Confinement-deconfinement interplay in quantum phases of doped Mott insulators

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It is generally accepted that doped Mott insulators can be well characterized by the t-J model. In the t-J model, the electron fractionalization is dictated by the phase string effect. We found that in the underdoped regime, the antiferromagnetic and superconducting phases are dual: in the former, holons are confined while spinons are deconfined, and *vice versa* in the latter. These two phases are separated by a novel phase, the so-called Bose-insulating phase, where both holons and spinons are deconfined. A pair of Wilson loops was found to constitute a complete set of order parameters determining this zero-temperature phase diagram. The quantum phase transitions between these phases are suggested to be of non-Landau-Ginzburg-Wilson type.

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Introduction.–The concept of fractionalization, finding its analog in quantum chromodynamics, is nowadays a guiding principle of strongly correlated systems [1]. Specifically, a quasiparticle (like electron), at short spacetime scales, is effectively fractionalized into a few degrees of freedom (like spin and charge) which, at large scales, are "glued" by emergent gauge degrees of freedom. As such, the gauge fields affect profoundly the formation of various novel quantum phases [2]. A fundamental issue is the nature of the quantum phase transition between these unconventional phases. In the conventional Landau-Ginzburg-Wilson (LGW) symmetry-breaking paradigm, different phases are characterized by the presence or absence of certain local order parameters. On the contrary, the fluctuation of emergent gauge field may contribute to low energy excitations of the system. In general, the transition between such phases cannot be described by the symmetry breaking paradigm, as is exemplified in Ref. [3].

Practically, an important playground of fractionalization is doped cuprates [1, 2]. Despite the debate about the nature of the cuprate phase diagram, it is widely accepted that the essential physics of high T_c superconductivity is captured by a doped Mott insulator where the electron fractionalization into spinons and holons is seemingly inevitable [1, 2, 4, 5]. Indeed, there have been abundant evidence indicating that the electron fractionalization leads to new quantum phases [2] and their transitions may not be understood in terms of symmetry breaking. This opinion has been reiterated very recently in Ref. [6], where it was conjectured that the drastic change in the nature of quantum statistics–a direct manifestation of electron fractionalization–is at the root of the pseudogap phase found in the cuprates.

Many fundamental issues arise thereby. *How does the electron fractionalization turn an antiferromagnet into a superconductor upon doping? What does the phase diagram look like and what are the underlying order parameters?* Substantial efforts [7–11] suggest that an exact non-perturbative result, the so-called *phase string effect*, discovered for the t-J model [7] is at the core of these issues. In this Letter, we present an analytical study of these open issues based on this exact result. We stress that our theory may be extended to other systems such as the Hubbard model on the honeycomb lattice currently undergoing intense investigations [12].

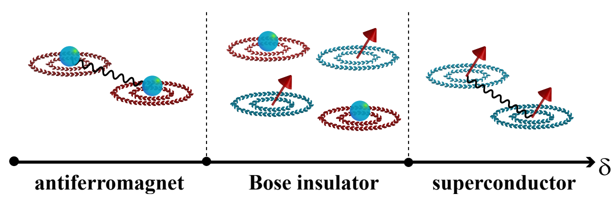

FIG. 1: (Color online) Zero temperature phase diagram of underdoped Mott insulators. The ball (in blue) and the arrow (in red) stand for the holon and spinon, respectively. The vortex, in red (blue), surrounding a holon (spinon) arises from the spinon (holon) condensate. The wavy line stands for the confinement.

Main results and qualitative discussions.–In essence, the phase string effect renders an electron "fractionalized" into two topological objects, the spinon and the holon, each of which is bosonic and carries a π-flux [13]. A $U(1)$ gauge field A^h_μ (A^s_μ), radiated by the holons (spinons), interacts with spinons (holons) through minimal coupling. (This is the so-called mutual statistical interaction which was also found in different contexts [12, 14]). The macroscopic electric current (density), $\mathbf{j}^h$, is fully carried by holons and driven by both the external electric field $\mathbf{E}$ and the "electric field" $\mathbf{E}^s$ resulting from A^s_μ induced by spinons. Interestingly, $\mathbf{E}^s$ finds its origin analogous to that of Ohmic dissipation in type-II superconductors: each spinon mimics a "magnetic vortex" suspending in holon fluids and, upon moving, generates an electric field *antiparallel* to $\mathbf{j}^h$, i.e., $\mathbf{E}^s = -\pi^2\sigma_s\mathbf{j}^h$, with

σ_s the spinon conductivity characterizing the mobility of spinons. From the Ohm's law, i.e., $\sigma_h^{-1}\mathbf{j}^h = -\pi^2\sigma_s\mathbf{j}^h + \mathbf{E}$, with σ_h the holon conductivity, we find that the (electric) resistivity, defined as $\sigma^{-1}\mathbf{j}^h = \mathbf{E}$, is

$$\sigma^{-1} = \sigma_h^{-1} + \pi^2\sigma_s. \tag{1}$$

A microscopic derivation of this composition rule will be given later.

For the doping δ sufficiently small, the antiferromagnetic (AF) phase is reached where the spinons are deconfined and form superfluids, i.e., $\sigma_s \to \infty$. According to Eq. (1) this phase is insulating, giving a vanishing σ. Indeed, single holon cannot appear in the excitation spectrum. Rather, holons are excited in pair and are logarithmically confined. These pairs are bound to the vortices of spinon superfluids and are thus immobile. Upon increasing δ, the typical size of holon pairs becomes larger and larger and, as a result, the confinement becomes weaker and weaker. Eventually, a quantum critical point (QCP) is reached. Beyond this QCP holons become deconfined. For sufficiently large δ, a dual scenario applies. The holons are deconfined and form superfluids, i.e., $\sigma_h \to \infty$. In this phase, single spinon cannot be excited. Rather, spinons are excited in pair, logarithmically confined and bound to the vortices of holon superfluids, i.e., $\sigma_s = 0$. (Consequently, the excitation spectrum is composed of integer spin excitations.) According to Eq. (1) this phase is superconducting (SC), i.e., $\sigma \to \infty$. Upon decreasing δ, the confinement becomes weaker and weaker. Eventually, another QCP is reached: for smaller δ spinons are deconfined. Thus, an intermediate phase can appear where both spinon and holon vortices are condensed. In this phase, $\sigma_{s,h} \to \infty$ implies $\sigma = 0$. This brings us to the term, Bose insulating (BI) phase.

The zero temperature phase diagram described above is summarized in Fig. 1. All three phases are characterized by confinement or deconfinement of spinons and holons, formally implemented by unconventional order parameters–a pair of Wilson loops $W_\delta^{s,h}[\mathcal{C}]$ with $\mathcal{C}$ a spacetime rectangle. Specifically, they are defined as the expectation values of $e^{i\oint_\mathcal{C} A_\mu^{s,h}dx^\mu}$ that probes the interaction between a pair of test holons (spinons). $W_\delta^s[\mathcal{C}]$ ($W_\delta^h[\mathcal{C}]$) displays nonanalyticity at the holon (spinon) deconfinement QCP. The existence of these nonlocal "order parameters" suggests the non-LGW nature of the quantum phase transitions: in contrast to an LGW type scenario of order parameter competing, the AF and SC phases are intrinsically incompatible such that they cannot be turned into unless confinement or deconfinement occurs. In particular, they are generally separated by the BI phase, with the AF (SC)-BI transition as a holon (spinon) deconfinement QCP. We now turn to present some technical details.

Lattice field theory.–Formally, we start from the Hamiltonian of the t-J model, which consists of the superexchange and hopping terms, describing the spin-flip and charge hopping process, respectively. An exact transformation [7] keeping track of the phase string effect transforms the superexchange term into ($\alpha = 1, 2$)

$$H_J \propto \sum_{i\alpha\sigma\sigma'} e^{-i\sigma A_\alpha^h(i)} b_{i-\sigma}^\dagger b_{i+\hat{\alpha}\sigma}^\dagger e^{i\sigma' A_\alpha^h(i)} b_{i+\hat{\alpha}\sigma'} b_{i-\sigma'}$$

and the hopping term into

$$H_t \propto \sum_{i\alpha\sigma} e^{iA_\alpha^s(i)} h_i^\dagger h_{i+\hat{\alpha}} e^{-i\sigma A_\alpha^h(i)} b_{i+\hat{\alpha}\sigma}^\dagger b_{i\sigma} + \text{h.c.},$$

with the coefficients omitted and h.c. being the Hermitian conjugate. Here, $\hat{\alpha} = \hat{x}, \hat{y}$ denotes the unit vector in the α-direction. The spinon ($b_{i\sigma}^\dagger, b_{i\sigma}, \sigma = \uparrow, \downarrow$) and holon ($h_i^\dagger, h_i$) operators are both bosonic. The mutual statistics obeyed by spinons and holons is accounted for the following topological constraints satisfied by the gauge fields. For each loop $\mathcal{C}$ in the lattice plane,

$$\begin{aligned} \textstyle\sum_{x\in\mathcal{C}} A_\alpha^s J_\mathcal{C}^\alpha &\equiv \pi \textstyle\sum_{x \text{ inside } \mathcal{C}} \left(b_\uparrow^\dagger b_\uparrow - b_\downarrow^\dagger b_\downarrow\right) \bmod 2\pi, \\ \textstyle\sum_{x\in\mathcal{C}} A_\alpha^h J_\mathcal{C}^\alpha &\equiv \pi \textstyle\sum_{x \text{ inside } \mathcal{C}} h^\dagger h \bmod 2\pi. \end{aligned} \tag{2}$$

Here, $J_\mathcal{C}^\alpha(x) = +1\,(-1)$ for the link $x \to x+\hat{\alpha}$ ($x+\hat{\alpha} \to x$) on $\mathcal{C}$, and is zero otherwise.

In the underdoped regime, it suffices to invoke the mean field approximation [7, 8], with the Hamiltonian simplified to $H = -\sum_{i\alpha}(J_s\sum_\sigma e^{-i\sigma A_\alpha^h(i)} b_{i-\sigma}^\dagger b_{i+\hat{\alpha}\sigma}^\dagger + t_h e^{i(A_\alpha^s(i)+A_\alpha^e(i))} h_i^\dagger h_{i+\hat{\alpha}}) + \text{h.c.}$ ($J_s, t_h > 0$). The external gauge field $A_\alpha^e(i)$ generates an electromagnetic field and couples to the holon degree of freedom. The Hamiltonian H and the constraints (2) constitute our exact starting point.

To proceed further, we consider the coherent state path integral representation of the partition function, $\mathcal{Z} = \operatorname{Tr} e^{-\beta H}$, where $\beta\,(\to \infty)$ is the inverse temperature. The technical challenge arises mainly from taking the topological constraints (2) into account. Such a task was fulfilled in Ref. [9] in the continuum limit. Here, we extend substantially the previous results to a more realistic lattice model. We are able to show $\mathcal{Z} = \iint_D e^{-\sum_x \mathscr{L}}$ with

$$\begin{aligned} \mathscr{L} &= \mathcal{L}_s + \mathcal{L}_h + V \\ &\quad + \frac{i}{\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu^s - 2\pi\mathscr{N}_\mu^s\right) d_\nu \left(A_\lambda^h - 2\pi\mathscr{N}_\lambda^h\right). \end{aligned} \tag{3}$$

Here, $\mathscr{N}_\mu^{s,h}$ are integer fields and $\mathcal{L}_{s,h}$ is the spinon and holon Lagrangian, respectively,

$$\mathcal{L}_s = (b_\uparrow^\dagger, b_\downarrow) \begin{pmatrix} D_{0\uparrow}^h + \lambda^s & -J_s D_\alpha^h \\ -J_s D_\alpha^h & D_{0\downarrow}^h + \lambda^s \end{pmatrix} \begin{pmatrix} b_\uparrow \\ b_\downarrow^\dagger \end{pmatrix},$$

$$\mathcal{L}_h = h^\dagger \left(D_0^s + \lambda^h - t_h D_\alpha^s\right) h.$$

Here and below, the Einstein's summation convention is implied and the summation over the indices μ, ν, λ includes both the imaginary time and spatial components. The on-site repulsive potential V softens the

hard-core boson condition and depends on $h^\dagger h$ and $b_\sigma^\dagger b_\sigma$. We shall not further present its details to which the results below are insensitive. $\epsilon^{\mu\nu\lambda}$ is the totally antisymmetric tensor. Finally, the notation "$\iint_D$" stands for the integral over the fields: $b^\dagger, b, h^\dagger, h, A^{s,h}$ and the Lagrange multipliers $\lambda^{s,h}$, and the summation over $\mathscr{N}_\mu^{s,h}$, $D_\alpha^s = e^{i(A_\alpha^s + A_\alpha^e)} e^{-d_\alpha} + e^{d_\alpha} e^{-i(A_\alpha^s + A_\alpha^e)}$, $D_\alpha^h = e^{iA_\alpha^h} e^{-d_\alpha} + e^{d_\alpha} e^{-iA_\alpha^h}$, $D_0^s = d_0 - iA_0^s$, $D_{0\sigma}^h = \sigma(d_0 - iA_0^h)$ with d_μ the lattice derivative. With these preparations, the pair of order parameters are defined as

$$W_\delta^{s,h}[\mathcal{C}] \equiv \mathcal{Z}^{-1} \iint_D e^{-\sum_x (\mathscr{L} - iA_\mu^{s,h} J_\mathcal{C}^\mu)}, \tag{4}$$

where $\mathcal{C}$ is a spacetime rectangle with length T (R) in the imaginary time (spatial) direction and $T \gg R$.

Differing from the prototypical field theory [9], $\mathscr{L}$ keeps firm track of the compact nature of $A_\alpha^{s,h}$, which affects profoundly the ground state properties, as will be shown below. In particular, the *lattice mutual Chern-Simons term*, namely the last term in Eq. (3) (Such a term was found previously in a study of Josephson junction arrays [15].), is periodic under a shift: $A_\alpha^{s,h} \to A_\alpha^{s,h} + 2\pi m_\alpha^{s,h}, \mathscr{N}_\alpha^{s,h} \to \mathscr{N}_\alpha^{s,h} + 2\pi m_\alpha^{s,h}$ with $m_\alpha^{s,h} \in \mathbb{Z}$. Moreover, summing up $\mathscr{N}_0^{s,h}$ enforces $\epsilon^{0\mu\nu} d_\mu A_\nu^{h,s}$ to be $m\pi$ with $m \in \mathbb{Z}$.

Superconducting phase.–Consider the case of dilute spin excitations where we may ignore the spinon field, i.e., $b^\dagger = b = 0$. Then, $\lambda^h |h|^2 + V$ gives rise to the holon superfluid. More precisely, factorizing the holon field as $h(x) = |h| e^{i\theta(x)}$ and inserting it into $\mathcal{L}_h + V$, we find that the fluctuation of $|h|$ is massive, whereas the Goldstone mode $\theta(x)$ is massless. Therefore, we ignore the terms associated with the spatial fluctuations of $|h|$ and obtain $\mathcal{L}_h = i|h|^2 (d_0\theta - A_0^s - i\lambda^h) - 2t_h |h|^2 \sum_\alpha \cos(d_\alpha \theta - A_\alpha^s)$ in the absence of A_α^e, which is further simplified to $i|h|^2 (d_0\theta - A_0^s - i\lambda^h) + t_h |h|^2 [(d_\alpha\theta - A_\alpha^s)^2 - 2]$. (By the definition of H, a 2π-shift in $d_\alpha\theta$ is absorbed into A_α^s.)

To calculate $W_\delta^h[\mathcal{C}]$, we separate A^h into the background value and the fluctuation. The former leads to a uniform flux, $\pi\delta$, at each plaquette and does not contribute to $W_\delta^h[\mathcal{C}]$. Then, we introduce the unitary gauge so as to incorporate $d_\mu\theta$ into A_μ^s, and insert the simplified expression of $\mathcal{L}_h$ into $\mathscr{L}$. Integrating out the matter and A^s fields, we find

$$\begin{aligned} W_\delta^h[\mathcal{C}] \sim \sum_{\{\mathscr{N}^s\}} \int D(a^h) e^{-\frac{1}{4}\sum_x F_{\mu\nu}^h F^{h\,\mu\nu}} \\ \times e^{i\pi\sqrt{2t_h\delta}\sum_x a_\mu^h (J_\mathcal{C}^\mu + 2\epsilon^{\mu\nu\lambda} d_\nu \mathscr{N}_\lambda^s)} \end{aligned} \tag{5}$$

upon appropriate rescaling, where $F_{\mu\nu}^h = d_\mu a_\nu^h - d_\nu a_\mu^h$ is the Maxwell tensor with a^h the fluctuating component of A^h, and $\sqrt{2t_h\delta}$ is the bare "charge". In the subsequent step, we integrate out a^h by using the Feynman gauge, which leads to important consequences. First of all, we find that at the ground state (where the external source $\pi J_\mathcal{C}^\mu$ is absent), two phase vortices of the holon superfluid carrying opposite vorticity $\pm 2\pi\epsilon^{0\mu\nu} d_\mu \mathscr{N}_\nu^s$ are logarithmically confined. Therefore, no free phase vortices exist with $\mathscr{N}_\alpha^s$ set to zero. Then, $\pi J_\mathcal{C}^\mu$ mimics an external dipole which may be produced by a pair of excited spinons with identical or opposite spin polarizations. In the latter case, a vortex with a vorticity of -2π is excited from the background and bound to a spinon, forming a dipole. Taking these considerations into account, we find

$$\ln W_\delta^h[\mathcal{C}] \sim -\pi t_h \delta\, T \ln R, \tag{6}$$

which shows that the spinons are logarithmically confined. To calculate $W_\delta^s[\mathcal{C}]$, we ignore a_α^h. Integrating out the matter fields, we find the holon deconfinement,

$$\begin{aligned} \ln W_\delta^s[\mathcal{C}] &\sim \ln \int D(A^s)\, e^{-t_h\delta \sum_x A_\mu^s A^{s\mu} + i\sum_x A_\mu^s J_\mathcal{C}^\mu} \\ &\sim -\frac{1}{4t_h\delta}(T+R). \end{aligned} \tag{7}$$

To further probe the SC long range order, we consider the response to a static small magnetic field. For this purpose we need to substitute A_α^s by $A_\alpha^s + A_\alpha^e$ in $\mathcal{L}_h$. If $A_\alpha^e = 2m\pi$ ($m \in \mathbb{Z}$), the partition function $\mathcal{Z}$ remains unchanged. For $A_\alpha^e = (2m+1)\pi$, the additional π-phase may be absorbed into A_α^s, leaving $\mathcal{L}_h$ (and thereby $\mathcal{Z}$) unaffected. According to Eq. (2), we conclude that a single spin, with two possible polarization directions, is locally excited and nucleated at the magnetic vortex core [10]. As such, the external magnetic field $m\pi$ flux is fully screened–a profound result of the integer field $\mathscr{N}_0^h$. If the flux value is not $m\pi$, the magnetic field is excluded by the superconductor. Indeed, in this case, $\mathcal{Z}$ is merely determined by A^e modulo π (denoted as $\tilde{A}^e$), $\mathcal{Z} \sim \int D(\theta)\, e^{2t_h\delta\beta \sum_{i\alpha} \cos\left(d_\alpha\theta - \tilde{A}_\alpha^e\right)} \sim e^{-t_h\delta\beta \sum_\mathbf{q} |\tilde{A}_\perp^e(\mathbf{q})|^2}$, which justifies the Meissner effect. Here $\tilde{A}_\perp^e(\mathbf{q})$ is the transverse part of the Fourier transform of $\tilde{A}^e$.

Antiferromagnetic phase.–We turn now to the case where holons are dilute and, likewise, set $h^\dagger = h = 0$. Then, $\lambda^s(|b_\uparrow|^2 + |b_\downarrow|^2) + V$ gives rise to a two-component spinon superfluid, with $|b_\uparrow| \approx |b_\downarrow| \approx \sqrt{n}$ implying a magnetization in the transverse direction. Here n is the concentration of condensed spinons depending on δ. Similar to the discussions on the SC phase, we factorize the two-component spinon field, $(b_\uparrow^\dagger(x), b_\downarrow(x))$, as $(|b_\uparrow|, |b_\downarrow|) e^{-i\theta(x)}$ and insert it into $\mathcal{L}_s + V$. Ignoring the spatial fluctuations of $|b|$, we obtain $\mathcal{L}_s = \sum_\sigma |b_\sigma|^2 [i(\sigma d_0\theta - \sigma A_0^h - i\lambda^s) - 2J_s \sum_\alpha \cos(d_\alpha\theta - A_\alpha^h)]$. Integrating out the matter and A^h fields gives

$$\begin{aligned} W_\delta^s[\mathcal{C}] \sim \sum_{\{\mathscr{N}^h\}} \int D(A^s) e^{-\frac{1}{4}\sum_x F_{\mu\nu}^s F^{s\,\mu\nu}} \\ \times e^{i\pi\sqrt{4nJ_s}\sum_x A_\mu^s (J_\mathcal{C}^\mu + 2\epsilon^{\mu\nu\lambda} d_\nu \mathscr{N}_\lambda^h)}, \end{aligned} \tag{8}$$

where $F_{\mu\nu}^s = d_\mu A_\nu^s - d_\nu A_\mu^s$.

Eq. (8) has far-reaching consequences. First of all, by integrating out the A^s field, we find that at the ground state ($\pi J^\mu_{\mathcal{C}} = 0$) the phase vortices of the spinon superfluid carrying opposite vorticity $\pm 2\pi\epsilon^{0\mu\nu}d_\mu \mathscr{N}^h_\nu$ are logarithmically confined. Most importantly, $\pi J^\mu_{\mathcal{C}}$ mimics an external dipole produced by a pair of the holon and anti-holon. The latter is a $-\pi$-fluxoid, formed out of a -2π phase vortex and a π-flux carried by the holon. Such a holon-anti-holon pair is logarithmically confined,

$$\ln W^s_\delta[\mathcal{C}] \sim -2\pi J_s n\, T \ln R. \qquad (9)$$

In other words, a holon pair nucleates in a phase vortex of the spinon superfluid of vorticity -2π, forming a "neutral" object. Furthermore, since the phase vortex is static, the pair of holons is spatially localized and the AF phase is insulating (see below for further explanations). It should be noticed that without the integer field $\mathscr{N}^h_\mu$, such an insulating phase cannot be established. Instead, the SC phase is pushed all the way to $\delta = 0$ [8]. Repeating the derivation of Eq. (7), we find the spinon deconfinement,

$$\ln W^h_\delta[\mathcal{C}] \sim -\frac{1}{8J_s n}(T + R). \qquad (10)$$

Bose insulating phase.–The analytic results, namely Eqs. (6), (7), (9) and (10), allow us to make an important observation. The asymptotic behavior, Eqs. (6) and (10), signal a critical concentration separating the spinon confinement and deconfinement phases, at which $W^h_\delta[\mathcal{C}]$ is nonanalytic in δ. Indeed, in the SC phase, the spinon excitations become progressively important as δ decreases: they cause a renormalization of the bare "charge" in Eq. (6) and eventually drive the system to the spinon deconfinement QCP where the "charge" vanishes. For smaller δ the SC long range order disappears. Likewise, Eqs. (7) and (9) signal another critical concentration, at which $W^s_\delta[\mathcal{C}]$ is nonanalytic, separating the holon confinement and deconfinement phases. Renormalizing the bare "charge" in Eq. (9) drives the system towards this QCP. For δ larger than this critical value, the AF long range order disappears. Because these two renormalization mechanisms are independent, these two QCPs are generally not identical, giving an intermediate phase where both the AF and SC long range orders vanish and both the holon and spinon are deconfined (condensed).

This intermediate phase does not support dc electric transport. To prove this, we notice that the composition rule (1) is valid for the entire underdoped regime. Indeed, minimizing $\mathscr{L}$ gives $\frac{\delta \mathcal{L}_{s,h}}{\delta A^{h,s}_\alpha} = -\frac{i}{\pi}\epsilon^{\alpha\mu\nu}d_\mu(A^{s,h}_\nu - 2\pi\mathscr{N}^{s,h}_\nu)$, i.e., $j^{s,h}_\alpha = \frac{1}{\pi}\epsilon^{0\alpha\beta}E^{s,h}_\beta$. By definition, $j^s_\alpha \equiv \sigma_s E^h_\alpha$ and $j^h_\alpha \equiv \sigma_h(E^s_\alpha + E_\alpha)$. Noticing that the electric and holon currents are identical, we obtain Eq. (1) from these three relations. The striking structure of the composition rule is a consequence of i) that the two pieces involved are vortices, and ii) that they obey mutual statistical interaction. (This fact has been established in a completely different context [14].) For the BI phase, $\sigma_s \to \infty$ because of spinon condensation, implying $\sigma = 0$.

Crossover from Mott's law to activation law.–Finally, we discuss a possible experimental observation of resistivity in the AF phase at sufficiently low but nonvanishing temperatures. In this regime, a holon-anti-holon pair bound to the vortex of spinon superfluids displays two-dimensional variable-range hopping conduction known as the Mott's law, $\ln \sigma_h^{-1} \sim T^{-1/3}$ [16]. On the other hand, spinon superfluid is actually the so-called strongly entangled vortex phase [17]. Carrying the arguments of Ref. [17] to the present context, we find that the spin transport, in response to $\mathbf{E}^h$, displays an activation law, i.e., $\ln \sigma_s \sim T^{-1}$. According to Eq. (1), the resistivity displays a crossover from Mott's law to activation law upon decreasing temperatures, which may serve as a probe of holon (spinon) confinement (deconfinement) in the AF phase.

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[1] Z. Zou and P. W. Anderson, Phys. Rev. B **37**, 627 (1988).
[2] For a review, see, P. A. Lee, N. Nagaosa, and X. G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[3] T. Senthil, *et. al.*, Science **303**, 1490 (2004).
[4] T. Senthil and M. P. A. Fisher, Phys. Rev. B **62**, 7850 (2000).
[5] S. Sachdev, Rev. Mod. Phys. **75**, 913 (2003).
[6] J. Zaanen and B. J. Overbosch, arXiv:0911.4070.
[7] Z. Y. Weng, *et. al.*, Phys. Rev. B **55**, 3894 (1997).
[8] See, for a review, Z. Y. Weng, Intl. J. Mod. Phys. B **21**, 773 (2007).
[9] S. P. Kou, X. L. Qi, and Z. Y. Weng, Phys. Rev. B **71**, 235102 (2005).
[10] V. N. Muthukumar and Z. Y. Weng, Phys. Rev. B **65**, 174511 (2002).
[11] J. W. Mei and Z. Y. Weng, Phys. Rev. B **81**, 014507 (2010).
[12] C. Xu, Phys. Rev. B **83**, 024408 (2011).
[13] The spacetime lattice constant, the Planck's constant, the speed of light in vacuum, and the electron charge are set to unity.
[14] V. M. Galitskii, *et. al.* Phys. Rev. Lett. **95**, 077002 (2005).
[15] M. C. Diamantini, *et. al.*, Nucl. Phys. B **474**, 641 (1996).
[16] B. I. Shklovskii and A. L. Efros, *Electronic properties of doped semiconductors* (Springer, Berlin, 1984).
[17] M. V. Feigelman, *et. al.* Phys. Rev. B **48**, 16641 (1993).